Discussion

of

"Indeterminacy, Causality, and the Foundations of Monetary Policy Analysis"
by Bennett T. McCallum

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Rational expectations models often yield several dynamically stable solutions. This property, however, is highly unwelcome if a model is to be used to describe actual economic behavior or to provide policy recommendations. When multiple equilibria arise, the economist is left with the problem which of the possible solutions should be regarded as a valid description of the most likely outcome of, for instance, a specific policy measure.

The literature, to which Bennett McCallum has contributed extensively, has suggested several criteria to choose between different equilibria. McCallum (1983) proposes the minimum-state-variable criterion (MSV), which rules out equilibria in which variables that do not appear in a model's structural equations play a role, only because agents believe so (so-called sun-spot equilibria). In certain instances, however, agents in fact may believe in irrelevant variables and it is debatable whether such equilibria should be ruled out *a priori*. The notion of expectational stability, developed by DeCanio (1979) and Evans (1985), implies that only those equilibria are relevant to which a system returns after a small deviation of expectations from rationality. Learnability of the equilibrium, which has been explored in the work of Evans and Honkapohja (1991, 2001), is another attractive concept because in practice an equilibrium will only be attained if agents are able to "learn" their expectations from past outcomes. But this requirement is difficult to verify and depends on the way how learning is modeled. To sum up, all these approaches have in common that they cannot easily be applied to all sorts of models and examples can be constructed that fulfill one criterion but do not obey one of the others.

The present paper takes a different tack by stating that indeterminacy does not reflect "a multiplicity of solutions for a single model, but instead a multiplicity of models each with a single solution". The problem is illustrated with a univariate example:

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$$y_t = aE_t y_{t+1} + cy_{t-1}. \tag{1}$$

This model contains both feedback and expectational elements. Its solution is of the form $y_t = \phi y_{t-1}$, giving rise to a quadratic equation with the two roots

$$\phi^{(-)} = \frac{1-\sqrt{1-4ac}}{2a} \quad \text{and} \quad \phi^{(+)} = \frac{1+\sqrt{1-4ac}}{2a}.$$

How to choose between these two solutions? McCallum recommends to check the continuity of the solution coefficients with respect to the model parameters. When letting $a \to 0$, $\phi^{(+)}$ approaches plus and minus infinity whereas $\phi^{(-)}$ converges against c, showing that $\phi^{(-)}$ is continuous whereas $\phi^{(+)}$ is not. Interestingly, this condition also rules out sun-spot equilibria. Moreover, it turns out to lead to the same conclusions as McCallum's MSV criterion. If $c = 0$, y_{t-1} cannot be a state variable according to equation (1). But in that case $\phi^{(+)}$ would equal $1/a$, suggesting that y_{t-1} is part of the solution, which is obviously wrong.

By checking the continuity of the solution coefficients, the present paper develops an elegant and general method to choose between equilibria. Moreover, it is an appealing condition since one would not expect the solution to change radically if small changes in the model parameters occur. MCCALLUM (2009) shows that this univariate example extends to the multivariate case where continuity can be checked by computing the limit of the solution coefficient matrix when the coefficient matrix on the future expected endogenous variables approaches zero.

Naturally, the question arises what the significance of the second solution, $\phi^{(+)}$, is. The paper's answer is that we have to think of $\phi^{(+)}$ as the solution to a different model containing only inertial but no expectational influences, i.e., $y_{t+1} = (1/a)y_t + (c/a)y_{t-1}$. In this case the discontinuity of $\phi^{(+)}$ is irrelevant since $a \to 0$ is not part of the admissible parameter space – it would imply that the model's coefficients explode. In addition, when $c \to 0$ in this model we still have the influence from y_t on y_{t+1}, which is exactly what $\phi^{(+)}$ implies.

While continuity of the coefficients is a purely mathematical condition, McCallum also offers an intuitive interpretation by referring to the direction of causality. When the model is solved, taking into account the endogeneity of expectations, we obtain equilibrium conditions that are silent about the direction of causality.[1] But when setting up the model, we had in mind that equation (1)

1 See also COCHRANE (2007, p. 15) in his response to WOODFORD (2003a).

determines how y_t reacts to its past realizations and its future expectations. Causality in this model runs from y_{t-1} to y_t and from y_{t-1} to y_{t+1}, but is bidirectional between y_t and y_{t+1} since both variables are determined jointly. By contrast, the alternative model has a unidirectional causal structure from y_t to y_{t+1}. The solution $\phi^{(-)}$ is compatible with the bidirectional causality structure, while $\phi^{(+)}$ reflects unidirectional causality.

A final question concerns whether economists should worry about indeterminacy. Some authors consider indeterminacy as a challenge to policy makers because these multiple equilibria represent several, equally likely, possible outcomes for the economy being modeled. Consequently, policies that could lead to indeterminate equilibria should be avoided.[2] McCallum has taken an opposite view, arguing that the problem lies in one's understanding of the way how the economy is modeled and not in the actual economy itself, i.e., that "… [rational expectations] solution multiplicity should be viewed basically as a mathematical curiosity, stemming from an insufficiently specific definition of rational expectations, rather than as a substantive problem for actual policy makers" (McCallum 2003, p. 1173). The present paper is a further step in a research agenda that endeavors to show that "… 'deeper theorizing' is likely to dispose of the problem" (McCallum 1983, p. 140).

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2 See the discussion in Woodford (2003b).

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