Should Marginal Cost of Public Funds Include the Revenue Effect?

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1. Introduction

Pigou (1947, p. 34) claims that the social marginal cost of the public good is higher than its private marginal cost when the funding to cover production costs is raised through distortionary taxes, and hence the second-best level of public good provision is lower than the first-best level. This conjecture is important since asymmetric information or other constraints rule out person-specific lump-sum taxes, and public revenues thereby are typically raised by distortionary taxes (Christiansen, 2007, p. 26). Moreover, as a matter of fact, public sector activities comprise a considerable share of output. For example, the share of total government outlays in GDP averaged slightly above 40% for the OECD countries during 2002 (Aronsson, 2008, p. 350). Therefore, not surprisingly, Feldstein (1997, p. 197) and Ng (2000, p. 253) note that the central public finance question facing any country is the appropriate size of its government, which is meant to provide public goods like national defense, a legal system and police protection and so on.[1]

There is a tremendous literature on Pigou's conjecture since the early seventies, prompted by the advancements in the theory of optimal taxation. Despite being a

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1 Karras (1996, p. 193) notes that the debate about the optimal size of the public sector is one of the oldest and most enduring in economics.

long-standing and central issue in public finance, the issue appears to be an unresolved one (Slemrod and Yitzhaki, 2001, p. 189; Christiansen, 2007, p. 25). In a survey, Ballard and Fullerton (1992, p. 118) state that "In recent years, public finance economists have produced a large literature on the indirect cost of the tax system, and the implications for government expenditure … In virtually every case, however, much of the work boils down to an attempt to identify the MCF," where the MCF is the "marginal cost of public funds."

The MCF measures the loss incurred by society in raising an additional dollar of tax revenue to finance government spending, i.e., it is the social marginal cost divided by the private marginal cost. As emphasized by Dahlby (2008, p. 2), the MCF gives us "a very intuitive way of describing fiscal choices, and therefore it can be readily used to convey economists' insights to policy makers and the general public." In general, the MCF can give us insights into the optimal government size. Moreover, Dahlby (2008) demonstrates that, in practice, the MCF can be used to evaluate tax reforms, public expenditure programs, and other public policies, ranging from tax enforcement to privatization of public enterprizes.

Unfortunately, there are many different measures of the MCF. Dahlby (2008, p. 2) notes that "While a substantial literature on the MCF has developed over the last twenty years, much of this literature is fragmented because authors have used different measures for the MCF." Ballard (1990, p. 263) and Ballard and Fullerton (1992, p. 124) argue that, in order to classify measures of the MCF, it is important to make the distinction between two approaches. The so-called Pigou-Harberger-Browing approach assumes that tax revenue collections are held constant and the consumer is compensated by a lump-sum transfer. Hence, income effects wash out and only substitution effects remain (e.g., see Browning, 1976, p. 284). In contrast, in the so-called Stiglitz-Dasgupta-Atkinson-Stern approach, the spending level is changed and the level of distortionary taxation is changed correspondingly. This approach emphasizes the revenue effect, an income effect.[2]

Snow and Warren (1996) reconcile the above two approaches since their results imply that it depends upon the specific situation whether the MCF should take into account the revenue effect. This is because there is a project-specific

2 Another difference among the measures for the MCF is to use "compensation variation" or "equivalent variation" to measure "lost real income to the consumer": e.g., Stuart (1984) utilizes the equivalent variation whereas Ballard, Shoven, and Whalley (1985) and Browning (1987) utilize the compensation variation. However, Fullerton (1991, p. 303) shows that Stuart's measure is always very close to that of Ballard, Shoven, and Whalley (1985), and concludes that the choice among compensation variation and equivalent variation matters for total excess burden, but not for marginal excess burden.

effect, which depends on the complementarity or substitutability between a particular project and the existing set of taxed activities.[3] For example, if we build a road, and if the road encourages consumption of gasoline, and if gasoline is taxed, then the road helps to pay for itself through the gasoline tax. As a result, it would be more likely for the road to be approved than we might have suspected on the basis of the simple Samuelson condition.

This paper tries to reconcile the two approaches from a general viewpoint. This is because, as emphasized by Besley and Jewitt (1991, p. 1769), "governments do typically seem to operate in a decentralized manner, with different authorities responsible for taxing and spending decisions." Therefore, the authority responsible for taxing decision may not know the specific characteristics of the public good, and hence the authority needs general insights in evaluating the relative merits of the two approaches.

Our theory is a cross between the two approaches. On the one hand, because good economic analysis calls for joint consideration of both the expenditure and tax sides of public finance (Slemrod and Yitzhaki, 2001, p. 189), we follow Atkinson and Stern (1974) to assume that the optimal level of public expenditure and the optimal set of distortionary taxes are chosen simultaneously and the economy thereby is at its second-best optimum. On the other hand, we follow the idea of the Pigou-Harberger-Browing approach to derive a measure which aims at capturing the distortionary effect associated with commodity as opposed to lump-sum taxation. [We show that, in the measure proposed by Atkinson and Stern (1974), the comparison is with a "windfall" to the government.]

Section 2 sets up the model. In Section 3, we derive the optimality condition for second-best public good provision. We then use this optimality condition to derive the two traditional measures of the MCF – one is with the revenue effect and the other is without. Section 4 begins with utilizing the optimality condition derived in Section 3 to demonstrate that it depends upon the relation between the public and taxed private goods whether the revenue effect should be included in the MCF or not, a result similar to those of Snow and Warren (1996). However, the main purpose of Section 4 aims at deriving the new measure of the MCF. Section 5 concludes this paper.

3 Stuart (1984, p. 353) argues that "the equilibrium level of tax revenue generally depends on the way in which the government spends the revenue, the value of marginal excess burden cannot itself be independent of the type of the marginal spending." Ballard and Fullerton (1992, p. 125) also emphasizes this rule: "The MCF depends on the entire interaction between the public expenditure and taxed activities." Besley and Jewitt (1991, p. 1769) also emphasizes this rule in a different context.

2. The Model

The model is a standard model where the government follows the Ramsey commodity taxation rule to collect tax revenues to finance the provision of a public good. There are n taxed private commodities $x_1,\ldots,x_n$ with post-tax prices $p_1,\ldots,p_n$. The specific tax on the i-th private good is t_i, i.e., $t_i \equiv p_i - c_i$ where c_i is the pre-tax price of x_i. It is assumed that each pre-tax price is constant. The $\mathbf{x}$, $\mathbf{p}$ and $\mathbf{c}$ denote $(x_1,\ldots,x_n)'$, $(p_1,\ldots,p_n)'$ and $(c_1,\ldots,c_n)'$, respectively, where "$'$" stands for "transpose." Similarly, $\mathbf{t} \equiv \mathbf{p} - \mathbf{c}$.

There is a single consumer whose preference is represented by a direct utility function $U(\mathbf{x},z,g)$ where z represents the numeraire, and g is the level of public good provision. It is assumed that z is not taxed, and hence its post-tax price is equal with 1, a standard assumption in the literature (ATKINSON and STIGLITZ, 1980, pp. 371–372; STERN, 1986, p. 298). The public good has a total cost function $C(g)$.

The notation is defined as follows:

$V(\mathbf{p},g,y) \equiv \max_{\mathbf{x},z} U(\mathbf{x},z,g)$ s.t. $\mathbf{p}'\mathbf{x} + \mathrm{z} \leq y$, i.e., it is the indirect utility function where y is the lump-sum income.

$X^i(\mathbf{p},g,y) \equiv$ the Marshallian demand function for good x_i $(i = 1,\ldots,n)$.

$\mathbf{X}(\mathbf{p},g,y) \equiv (X^1(\mathbf{p},g,y),\ldots,X^n(\mathbf{p},g,y))'$.

$R(\mathbf{p},g,y) \equiv (\mathbf{p} - \mathbf{c})'\mathbf{X}(\mathbf{p},g,y)$, i.e., it is the tax revenue function.

If a capitalized notation has a subscript, then this subscript represents a partial differentiation. For example, we write V_y, V_i and X^i_j for $\partial V / \partial y$, $\partial V / \partial p_i$ and $\partial X^i / \partial p_j$, respectively.

$\alpha \equiv V_y$, i.e., it is the marginal utility of income.

$E(\mathbf{p},g,u) \equiv \min_{\mathbf{x},z} \mathbf{p}'\mathbf{x} + z$ s.t. $U(\mathbf{x},z,g) \geq u$, i.e., it is the expenditure function where u is the utility level.

$H^i(\mathbf{p},g,u) \equiv E_i$, i.e., it is the Hicksian demand function for x_i $(\mathrm{i} = 1,\ldots,n)$.

$\mathbf{H}(\mathbf{p},g,u) \equiv (H^1(\mathbf{p},g,u),\ldots,H^n(\mathbf{p},g,u))'$.

$W(\mathbf{p},g,u) \equiv -E_g$, i.e., it is the marginal willingness to pay for the public good.

$\partial \mathbf{X} / \partial \mathbf{p}'$ is the $n \times n$ matrix $[X^i_j]$. $\partial \mathbf{H} / \partial \mathbf{p}'$ is defined similarly.

3. The Optimality Conditions and the MCF

In Subsection 3.1, we derive the optimality condition for second-best public good provision. We present this optimality condition in two equivalent forms, one in terms of Marshallian demand functions, the other in terms of Hicksian demand functions. In Subsection 3.2, we present the two measures of the MCF, which correspond to the two forms of the optimality condition in Subsection 3.1. In Subsection 3.3, we also use the two forms of the optimality condition to derive two measures of the MCF which are applicable even when taxes are not set optimally.

3.1 Two Forms of Second-Best Optimality Condition

The social planner's second-best problem can be represented by

$$V^*(T,f) \equiv \max_{\mathbf{p},g} V(\mathbf{p},g,-T) \text{ s.t. } \quad R(\mathbf{p},g,-T)+T+f \geq C(g) \qquad (3.1)$$

where T and f are given, T is a lump-sum tax, and f is a windfall received by the social planner (e.g., foreign aid). Let the Lagrangian function of (3.1) be

$$L = V(\mathbf{p},g,-T)+\lambda[R(\mathbf{p},g,-T)+T+f-C(g)]$$

where λ is a Lagrange multiplier. Partially differentiating L with respect to g and dividing $\partial L/\partial g = 0$ by α give the following equation [Eq. (3) of Atkinson and Stern (1974)]:

$$W = \frac{\lambda}{\alpha}\left[\frac{\mathrm{d}C(g)}{\mathrm{d}g} - \mathbf{t}'\mathbf{X}_g\right]. \qquad (3.2)$$

Partially differentiating L with respect to $\mathbf{p}'$, and applying the Roy's identity towards $\partial L/\partial \mathbf{p}' = 0$ yields the following equation:

$$\alpha \mathbf{X}' = \lambda \frac{\partial R}{\partial \mathbf{p}'} = \lambda\left[\mathbf{X}' + \mathbf{t}'\frac{\partial \mathbf{X}}{\partial \mathbf{p}'}\right]. \qquad (3.3)$$

Multiplying the above equation by $\mathbf{t}$ from the right and dividing it by λR yield an expression for α/λ:

$$\frac{\alpha}{\lambda} = 1 + \mathbf{t}' \frac{\partial \mathbf{X}}{\partial \mathbf{p}'} \mathbf{t} \times \frac{1}{R}. \tag{3.4}$$

It is next to use the Slutsky equation to express the optimality condition in terms of Hicksian demand functions. The Slutsky equation is

$$\frac{\partial \mathbf{X}}{\partial \mathbf{p}'} = \frac{\partial \mathbf{H}}{\partial \mathbf{p}'} - \mathbf{X}_y \mathbf{X}'.$$

Applying the Slutsky equation towards (3.3) yields

$$\mu \mathbf{H}' = \lambda \left[\mathbf{H}' + \mathbf{t}' \frac{\partial \mathbf{H}}{\partial \mathbf{p}'} \right] \tag{3.5}$$

where $\mu \equiv \alpha + \lambda R_y$ is the so called "social marginal utility of income" introduced by DIAMOND (1975, p. 338). Just as (3.3) leads to (3.4), (3.5) leads to the following equation:

$$\frac{\mu}{\lambda} = 1 + \mathbf{t}' \frac{\partial \mathbf{H}}{\partial \mathbf{p}'} \mathbf{t} \times \frac{1}{R}. \tag{3.6}$$

Drawing upon (11) of KING (1986) yields the following equation:

$$\mathbf{X}_g = \mathbf{H}_g + \mathbf{X}_y W. \tag{3.7}$$

By substituting (3.7) into (3.2) yields the following equation [Eq. (3.4a) of BATINA and IHORI (2005)]:

$$W = \frac{\lambda}{\mu} \left[\frac{\mathrm{d}C(g)}{\mathrm{d}g} - \mathbf{t}'\mathbf{H}_g \right], \tag{3.8}$$

which is a counterpart of (3.2).

3.2 Two Measures of the MCF and the Revenue Effect

As noted by BALLARD and FULLERTON (1992, p. 124), neither STIGLITZ and DASGUPTA (1971) nor ATKINSON and STERN (1974) use the "marginal cost of public funds" terminology. However, Atkinson and Stern state that "α is the marginal utility of income and λ is the social marginal cost of raising revenue" (p. 122).

This may explain why λ / α is sometimes referred to as the MCF (e.g., Batina and Ihori, 2005, p. 32). Therefore, we can utilize λ / α to define the following MCF:

$$\text{MCF}^u \equiv \frac{\lambda}{\alpha} = \frac{1}{1 + \mathbf{t}' \dfrac{\partial \mathbf{X}}{\partial \mathbf{p}'} \mathbf{t} \times \dfrac{1}{R}} \tag{3.9}$$

where the superscript "u" denotes "uncompensated" since this definition includes an income effect. Applying the Slutsky equation towards (3.4) yields another expression for α / λ [Eq. (6) of Atkinson and Stern (1974)]:

$$\frac{\alpha}{\lambda} = 1 + \mathbf{t}' \frac{\partial \mathbf{H}}{\partial \mathbf{p}'} \mathbf{t} \times \frac{1}{R} - R_y. \tag{3.4'}$$

Therefore,

$$\text{MCF}^u = \frac{1}{1 + \mathbf{t}' \dfrac{\partial \mathbf{H}}{\partial \mathbf{p}'} \mathbf{t} \times \dfrac{1}{R} - R_y}. \tag{3.9'}$$

The term $\mathbf{t}' \partial \mathbf{H} / \partial \mathbf{p}' \mathbf{t} / R$ is nonpositive since the Slutsky matrix $\partial \mathbf{H} / \partial \mathbf{p}'$ is semi-negative definite. Therefore, Atkinson and Stern point out that this term works in the direction of $\lambda / \alpha > 1$, and hence represents "the distortionary effect with which Pigou was concerned."

However, they note that Pigou overlooked the term $-R_y$, which is referred to as the "revenue effect" by them. Moreover, they utilize this term to challenge Pigou's conjecture by emphasizing that $-R_y$ may be positive. For example, if $n = 1$ (i.e., there is only one taxed commodity), x_1 is the "negative labor supply," and the leisure is a normal good, then $-t_1 > 0$ (a negative tax rate on leisure raising positive tax revenue) and $X_y^1 > 0$, implying that $-R_y > 0$. In other words, if the labor supply is subject to a linear tax, and the Marshallian labor supply curve is backward-bending, then MCF^u may be less than unity.[4]

Just as the optimality condition (3.3) leads to MCF^u, the optimality condition (3.5) leads to the following measure:

4 Please refer to Proposition 1 of Chang and Peng (2009) for sufficient conditions for guaranteeing that $\text{MCF}^u > 1$.

$$\text{MCF}^c \equiv \frac{\lambda}{\mu} = \frac{1}{1 + \mathbf{t}' \dfrac{\partial \mathbf{H}}{\partial \mathbf{p}'} \mathbf{t} \times \dfrac{1}{R}} \tag{3.10}$$

where the superscript "c" denotes "compensated." By contrast to MCF^u, MCF^c does not include the revenue effect, $-R_y$. It is known that $0 < \mu / \lambda \leq 1$.[5] It thus is guaranteed that $\text{MCF}^c \geq 1$.

3.3 When Taxes Are not Set Optimally

As noted by Dahlby (2008, p. 24), "optimal taxation is a goal that is seldom realized in practice." It thus is important to derive measures of the MCF for the case where taxes are not set optimally. In this subsection, we follow an idea of Dahlby (2008, Sec. 2.3) to use the optimality condition to derive this kind of measures. Rewrite (3.3) as the following [Eq. (2.2') of Chang and Peng (2009)]:

$$\frac{\lambda}{\alpha} = \frac{X^i}{R_i}, i = 1, \ldots, n. \tag{3.11}$$

This equation suggests that the following is a measure of the MCF [Eq. (7) of Slemrod and Yitzhaki (2001)]:

$$\text{MCF}_i^u = \frac{X^i}{R_i}, i = 1, \ldots, n. \tag{3.12}$$

This equation is similar with (2.23) of Dahlby (2008).[6]

Eq. (3.11) implies that if taxes are set optimally, then, as emphasized by Dahlby (2008, p. 26) and Chang and Peng (2009, p. 682), the marginal cost of raising revenue is the same for all tax sources: $\text{MCF}_1^u = \text{MCF}_2^u = \ldots = \text{MCF}_n^u = \text{MCF}^u$. The MCF_i^u still measures the monetary cost of raising an additional dollar of tax revenue from good i, even when taxes are not set optimally. For example, if we

5 It is clear that $\mu > 0$, implying that $1 - \mu / \lambda < 1$. Eq. (3.6) implies that $1 - \mu / \lambda \geq 0$ since the Slutsky matrix is semi-negative definite.

6 In our notation, Dahlby uses $-V_i / (\lambda R_i)$ to measure the monetary cost of raising an additional dollar of tax revenue from good i. According to the Roy's identity, $-V_i / \lambda = \alpha X^i / \lambda$.

find that $\text{MCF}_i^u > \text{MCF}_j^u$, then, as noted by Dahlby (2008, p. 26), a revenue-neutral tax reform – switching revenues from good i to good j – will improve the taxpayer's well-being.

We can derive the above measure by an alternative approach. It is a popular practice in the previous literature [e.g., please refer to Browning (1976, p. 285) or Wildasin (1984, p. 227)] to use the following intuitive approach to capture the MCF: "lost real income to the consumer" being divided by "the increment in tax revenue."[7] From the viewpoint of this intuitive approach, MCF_i^u indeed is a measure of the MCF since, according to the Roy's identity, X^i captures the lost real income to the consumer (namely, $X^i = -V_i / \alpha$) and, by definition, R_i represents the increment in revenue.

Note that here we establish the relationship between the measure derived by the theoretical framework of Atkinson and Stern (1974), namely, MCF^u, with a measure derived by the intuitive approach, namely, MCF_i^u.

We can use the same procedure to derive the compensated counterpart of MCF_i^u. Eq. (3.5) similarly leads to the following equation:

$$\frac{\lambda}{\mu} = \frac{X^i}{R_i^c}, i = 1, \ldots, n \tag{3.13}$$

where $R^c(\mathbf{p}, u) \equiv \mathbf{t}'\mathbf{H}(\mathbf{p}, u)$. Eq. (3.13) suggests that the following is a measure of the MCF:

$$\text{MCF}_i^c = \frac{X^i}{R_i^c}, i = 1, \ldots, n. \tag{3.14}$$

We can show that the Pigou-Harberger-Browning approach uses the above measure.[8]

7 The very recent literature also continues to utilize this concept, e.g., see (16) of Liu (2003, p. 1714). This intuitive approach has the following advantage: it can come up with a number for the MCF without assuming that the economy is necessarily at its second-best optimum.

8 In the model of Wildasin (1984), there is only one taxed commodity, and the "SMC" defined in (1) is exactly MCF_i^c.

4. A New Measure of the MCF

Snow and Warren (1996) demonstrate that it depends upon the specific situation which measure of the MCF is correct. It is next to demonstrate that the optimality framework in Section 3 straightforwardly yields a similar result.[9] Ballard and Fullerton (1992, p. 118) define the MCF to be the number which is used to multiply the direct marginal cost in the cost-benefit analysis of a public project:

$$\sum \mathrm{MRS} = \mathrm{MCF} \times \mathrm{MRT}.$$

If we define the MCF in this way, then we can utilize the optimality condition to evaluate measures of the MCF. First, because $\mathrm{d}C(g)/\mathrm{d}g$ is the MRT, from (3.2) it follows that MCF^u is a correct measure of the MCF if $\mathbf{X}_g = 0$. Second, from (3.8) it follows that MCF^c is a correct measure of the MCF if $\mathbf{H}_g = 0$. Therefore, it depends upon the relation between g and $\mathbf{x}$ whether the revenue effect should be included in the MCF or not.

Both MCF^u and MCF^c include λ. It thus is important to analyze the essence of λ. According to (3.1), from the envelope theorem it follows that $\lambda = V_f^*$. This means that λ measures the marginal value of one dollar windfall, and hence λ/α measures the marginal monetary value of one dollar windfall, implying that MCF^u measures the social marginal cost of one dollar commodity tax revenue as opposed to one dollar windfall. In contrast, V_T^* is the *extra* value of one dollar lump-sum tax revenue being substituted for one dollar commodity tax revenue. Accordingly, the marginal monetary benefit of one dollar lump-sum tax revenue

9 Because Snow and Warren want to cover all the measures of the MCF in the literature, their setup is more general than ours. If we let $n = 1$, and let x_1 denote "negative labor supply," then their setup is similar with ours. However, in our setup, we assume (1) that there is no wage income exempt from taxation, and hence t_1 not only is a marginal tax rate but also is an average tax rate; (2) that all the tax revenue is utilized to finance the public good, i.e., in the terminology of Snow and Warren (1996, p. 292), the proportion of total tax revenue devoted to "exhaustive public spending" is one; (3) that the economy is situated in the second-best solution described in (3.1). Snow and Warren do not rely upon these assumptions. For example, like Mayshar (1990, 1991), they allow for a difference in average and marginal tax rates. Moreover, in their setup, there may be transfer payments, i.e., the proportion of total tax revenue devoted to "exhaustive public spending" may be less than one. Their (18) and (21) correspond to our (3.2) and (3.8). Notice that our equations, though much simpler, capture all the effects emphasized by them. For example, in their (18), there are "distortionary effect," "revenue effect" and "budget effect" (p. 296). It is clear that our framework also captures these three effects. [The second term on the right-hand side of (3.2) is the budget effect.]

being substituted for one dollar commodity tax revenue is $V_T^* / \alpha + 1$. In other words, MCF* measures the marginal monetary cost of one dollar commodity tax revenue being substituted for one dollar lump-sum tax revenue where

$$\mathrm{MCF}^* \equiv \frac{V_T^*}{\alpha} + 1. \tag{4.1}$$

From the envelope theorem it follows that

$$\frac{V_T^*}{\alpha} = -1 + \frac{\lambda}{\alpha}(1 - R_y). \tag{4.2}$$

Therefore,

$$\mathrm{MCF}^* = \frac{\lambda}{\alpha}(1 - R_y), \tag{4.3}$$

which, together with (3.9), implies that

$$\mathrm{MCF}^* = \mathrm{MCF}^u \times (1 - R_y). \tag{4.4}$$

Therefore, in practice, it is easy to obtain the new measure once MCF^u is obtained.

Substitute (3.4') into (4.3) to obtain

$$\mathrm{MCF}^* = \frac{1 - R_y}{1 + \mathbf{t}' \dfrac{\partial \mathbf{H}}{\partial \mathbf{p}'} \mathbf{t} \times \dfrac{1}{R} - R_y}. \tag{4.5}$$

Eq. (4.5) shows that MCF* involves the revenue effect. This is a surprising result since MCF* and MCF^c share the same basic idea, namely, both of them measure the social marginal cost of the commodity taxation as opposed to the lump-sum taxation.

Eq. (4.5) reveals that, compared with MCF^u in (3.9'), the MCF*, as expected, plays down the revenue effect since not only the denominator but also the numerator should be subtracted by R_y. Substituting (3.4') into (4.2) yields[10]

10 This equation is similar with (A.3) of Atkinson and Stern (1974, p. 127).

$$\frac{V_T^*}{\alpha} = -\frac{\lambda}{\alpha}\mathbf{t}'\frac{\partial \mathbf{H}}{\partial \mathbf{p}'}\mathbf{t}\times\frac{1}{R} \geq 0. \tag{4.6}$$

Therefore, from (4.1) it follows that MCF* should be larger than unity. This result is not surprising since MCF* plays down the revenue effect.[11]

We are in a position to find out the ranking between MCF^u, MCF^c and MCF*. We can use (4.4) to compare MCF* with MCF^u. The MCF* in (4.5) can be rewritten in the following form:

$$\text{MCF}^* = \frac{1}{1+\mathbf{t}'\dfrac{\partial \mathbf{H}}{\partial \mathbf{p}'}\mathbf{t}\times\dfrac{1}{(1-R_y)R}}.$$

Using this form, it is easy to compare MCF* with MCF^c. From $\mu/\lambda \leq 1$ it follows $R_y \leq 1-\alpha/\lambda < 1$. Therefore, it is without loss of generality to assume that are sure that $1-R_y > 0$. We can show that MCF* always lies between MCF^c and MCF^u:

$$\begin{cases} \text{MCF}^c < \text{MCF}^* < \text{MCF}^u, & \text{if } 0 < R_y < 1; \\ \text{MCF}^c > \text{MCF}^* > \text{MCF}^u, & \text{if } 0 > R_y. \end{cases}$$

The above inequalities also imply that if $R_y > 0$, then MCF^u overestimates the marginal cost of one dollar commodity tax revenue being substituted for one dollar lump-sum tax revenue while MCF^c underestimates. If $R_y < 0$,[12] then the opposite holds.

11 We are in a position to point out a neglected but important contradiction in ATKINSON and STERN (1974). As mentioned, they state that λ is the social marginal cost of raising revenue (p. 122), and emphasize that the revenue effect may let λ/α be lower than one. However, they also use the lump-sum taxation as a reference point when they mention the excess burden associated with commodity taxation; and, in particular, they exactly use V_T^* to capture the excess burden of commodity as opposed to lump-sum taxation (p. 123 and p. 127). This approach implies that the social marginal cost of raising revenue is $\lambda(1-R_y)$ since $V_T^* = \lambda(1-R_y)-\alpha$. Moreover, we have shown that if we let $\lambda(1-R_y)$ represent the social marginal cost, then its corresponding MCF is guaranteed to be higher than one, i.e., it is unambiguous that the revenue effect itself cannot overturn "Pigou's excess burden argument." Therefore, this approach itself suggests that Atkinson and Stern might overstate the significance of the revenue effect.

12 In the Ramsey taxation, a tax rate may be negative, especially when there are complementarities between the taxed goods (TIROLE, 1988, p. 70; BÖS, 1989, p. 202; CHANG and PENG,

5. Conclusions

There is no consensus in the literature as to how to measure the marginal cost of public funds. Most prominent are two competing proposals which can be traced back to the Pigou-Harberger-Browing approach and the Stiglitz-Dasgupta-Atkinson-Stern approach. The latter approach emphasizes the so called revenue effect, an income effect, whereas the former approach only involves substitution effects. The proposal of the Pigou-Harberger-Browing approach is the appropriate measure if the Hicksian demand for each taxed private commodity is independent of the public good provision level while the proposal of the Stiglitz-Dasgupta-Atkinson-Stern approach is the appropriate measure if the Marshallian demand for each taxed private commodity is independent of the public good provision level.

This paper tries to reconcile the two approaches from a general viewpoint since the authority responsible for taxing decision may not know the specific characteristics of the public good, and hence the authority needs general insights in evaluating the relative merits of the two approaches. We demonstrate that the revenue effect represents a distortionary effect associated with commodity taxation as opposed to a "windfall," instead of lump-sum taxation. Therefore, our research strategy is a cross between the two approaches. On the one hand, we follow the Stiglitz-Dasgupta-Atkinson-Stern approach to assume that the optimal level of public expenditure and the optimal set of distortionary taxes are chosen simultaneously and the economy thereby is at its second-best optimum. On the other hand, we follow the idea of the Pigou-Harberger-Browing approach to derive a measure which aims at capturing the distortionary effect associated with commodity as opposed to lump-sum taxation. We demonstrate that this new measure plays down, but still includes, the revenue effect. Hence, the new measure lies between the two measures proposed by the two approaches. Moreover, this new measure is guaranteed to be higher than unity.

2009, Proposition 2). Therefore, R_y is not guaranteed to be positive even each private taxed good is normal.

References

ARONSSON, THOMAS (2008), "Social Accounting and the Public Sector", *International Economic Review*, 49 (1), pp. 349–375.

ATKINSON, ANTHONY B., and NICHOLAS H. STERN (1974), "Pigou, Taxation and Public Goods", *Review of Economic Studies*, 41, pp. 119–128.

ATKINSON, ANTHONY B., and JOSEPH E. STIGLITZ (1980), *Lectures on Public Economics*, McGraw-Hill, New York.

BALLARD, CHARLES L. (1990), "Marginal Welfare Cost Calculations: Differential Analysis vs. Balanced-Budget Analysis", *Journal of Public Economics*, 41 (2), pp. 263–276.

BALLARD, CHARLES L., and DON FULLERTON (1992), "Distortionary Taxes and the Provision of Public Goods", *Journal of Economic Perspective*, 6 (3) pp. 117–131.

BALLARD, CHARLES L., JOHN B. SHOVEN and JOHN WHALLEY (1985), "General Equilibrium Computations of the Marginal Welfare Costs of Taxes in the United States", *American Economic Review*, 75, pp. 128–138.

BATINA, RAYMOND G., and TOSHIHIRO IHORI (2005), *Public Goods: Theories and Evidence*, Springer, New York.

BESLEY, TIMOTHY, and IAN JEWITT (1991), "Decentralizing Public Good Supply", *Econometrica*, 59 (6), pp. 1769–1778.

BÖS, DIETER (1989), *Public Enterprise, Economics Theory and Application*, 2nd ed., North-Holland, Amsterdam.

BROWNING, EDGAR K. (1976), "The Marginal Cost of Public Funds", *Journal of Political Economy*, 84, pp. 283–298.

BROWNING, EDGAR K. (1987), "On the Marginal Welfare Cost of Taxation", *American Economic Review*, 77, pp. 11–23.

CHANG, C., and HSIAO-PING PENG (2009), "Structure Regulation, Price Structure, Cross Subsidization and Marginal Cost of Public Funds", *The Manchester School*, 77 (5), pp. 618–641.

CHRISTIANSEN, VIDAR (2007), "Two Approaches to Determine Public Good Provision under Distortionary Taxation", *National Tax Journal*, 60 (1), pp. 25–43.

DAHLBY, BEV (2008), *The Marginal Cost of Public Funds: Theory and Applications*, MIT Press, Cambridge, Mass. and London.

DIAMOND, PETER A. (1975), "A Many-Person Ramsey Rule", *Journal of Public Economics*, 4, pp. 335–342.

FELDSTEIN, MARTIN (1997), "How Big Should Government Be?", *National Tax Journal*, 50 (2), pp. 197–213.

Fullerton, Don (1991), "Reconciling Recent Estimates of the Marginal Welfare Cost of Taxation", *American Economic Review*, 81 (1), pp. 302–308.

Karras, Georgios (1996), "The Optimal Government Size: Further International Evidence on the Productivity of Government Services", *Economic Inquiry*, 34 (2), pp. 193–203.

King, Mervyn A. (1986), "A Pigovian Rule for the Optimum Provision of Public Goods", *Journal of Public Economics*, 30, pp. 273–291.

Liu, Liqun (2003), "A Marginal Cost of Funds Approach to Multi-Period Public Project Evaluation: Implications for the Social Discount Rate", *Journal of Public Economics*, 87, pp. 1707–1718.

Mayshar, Joram (1990), "On Measures of Excess Burden and their Application", *Journal of Public Economics*, 41, pp. 263–289.

Mayshar, Joram (1991), "On Measuring the Marginal Cost of Funds Analytically", *American Economic Review*, 81, pp. 1329–1335.

Ng, Yew-Kwang (2000), "The Optimal Size of Public Spending and the Distortionary Cost of Taxation", *National Tax Journal*, 53 (2), pp. 253–272.

Pigou, Arthur C. (1947), *Public Finance*, Macmillan, London.

Slemrod, Joel, and Shlomo Yitzhaki (2001), "Integrating Expenditure and Tax Decisions: The Marginal Cost of Funds and the Marginal Benefit of Projects", *National Tax Journal*, 54 (2), pp. 189–201.

Snow, Arthur, and Ronald S. Warren, Jr. (1996), "The Marginal Welfare Cost of Public Funds: Theory and Estimates", *Journal of Public Economics*, 61, pp. 289–305.

Stern, Nicholas (1986), "A Note on Commodity Taxation: The Choice Of Variable and the Slutsky, Hessian and Antonelli Matrices (SHAM)", *Review of Economic Studies*, 53, pp. 293–299.

Stiglitz, Joseph E., and Partha Dasgupta (1971), "Differential Taxation, Public Goods and Economic Efficiency", *Review of Economic Studies*, 38 (114), pp. 151–174.

Stuart, Charles E. (1984), "Welfare Costs per Dollar of Additional Tax Revenue in the United States", *American Economic Review*, 74, pp. 352–362.

Tirole, Jean (1988), *The Theory of Industrial Organization*, MIT Press, Cambridge, Mass. and London.

Wildasin, David E. (1984), "On Public Good Provision with Distortionary Taxation", *Economic Inquiry*, 22, pp. 227–243.

SUMMARY

It is an important difference in different measures of the marginal cost of public funds whether to take into account the "revenue effect" emphasized by ATKINSON and STERN (1974). This paper tries to reconcile two competing measures from a general viewpoint. We demonstrate that the revenue effect represents a distortionary effect associated with commodity taxation as opposed to a "windfall." We thus derive a new measure which captures the distortionary effect associated with commodity as opposed to lump-sum taxation. This new measure is guaranteed to be higher than unity. Moreover, this new measure lies between the two measures.